# *University of Tripoli*
## *Faculty of Engineering*
## *Electrical and Electronics Engineering Department*


EE 303 Numerical Techniques and Programming

Midterm I, January 3rd, 2012

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a) Answer all questions to the best of your knowledge
b) Show all steps and carry all calculations up to 4 digits unless otherwise mentioned.
c) No question will be answered during the exam.
d) Time allowed: ***2 hours***

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**Q1-**

(a) Write the first 5 terms of Taylor series at 0 for the function $f(x) = \sqrt{x+1}$

(b) Using *f(0)*. How many terms are needed for the absolute error to drop down to $10^{-2}$, given the true value *f(0.2)*= 1.0954.

(c) What is the difference between round-off error and truncation error.

(5 Marks)

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**Q2-**

(a) Using Newton-Raphson's method, find the root of $f(x) = e^{-x} - 2x^2 + 2$, start with $x$=2.0 and repeat until the absolute error drops down to <= $10^{-7}$ *(carry out calculations to 6 digits)*

(b) Graphically derive the formula for the secant method and use it to find the root of the equation in part a of this question. Start with $x_0 = 2$ *and* $x_1 = 1$. *Perform only 4 iterations.*

(10 Marks)

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**Q3.**

Given the following system of equations

$$-3y + z = 16.1$$
$$2x + 6y - 4z = -49.0$$
$$-8x - 2y + 5z = 18.9$$

(a) Use forward gauss elimination with partial pivoting to solve the system.

(b) Solve the system using LU decomposition

(10 Marks)

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*Good luck to all of you*

*University of Tripoli*
*Faculty of Engineering*
*Electrical and Electronics Engineering Department*

EE 303 Numerical Techniques and Programming

Midterm II, February 7th, 2012

a) *Answer all questions to the best of your knowledge.*
b) *Show all steps and carry all calculations up to 4 digits unless otherwise mentioned.*
c) *No question will be answered during the exam.*
d) *Time allowed:* ***1.30 hours***

Q1- (a) Find the Euclidean norm of the following matrix

$$A = \begin{bmatrix} 10 & -5 & 3 \\ -5 & -10 & 1 \\ 3 & 7 & -9 \end{bmatrix}$$

(b) What is a value that will make the condition number of the following matrix $\approx 100$

$$A = \begin{bmatrix} 1 & 2 \\ 3 & a \end{bmatrix}$$

(5 Marks)

Q2- (a) Given $f(x) = \sqrt{1+x^2}$, show that the relative error is halved when $\Delta x$ is halved for forward and backward difference and quartered for centered difference differentiation.(use $x$=1 and $\Delta x$ = 0.1, 0.05 and 0.025).

(b) Use Lagrange polynomial of 3rd degree to find $f(5.5)$ from the following table

| | | | | | | |
|---|---|---|---|---|---|---|
| $x$ | 1.1 | 4.3 | 6.2 | 8.0 | 9.5 | 10.0 |
| $f(x)$ | -6.570 | 6.87 | 43.92 | 99.0 | 159.75 | 183.0 |

(10 Marks)

Q3- (a) Use Divided Difference Polynomial to derive Simpson's h/3 rule of numerical Integration using only 3 points ($x_0, x_1$ *and* $x_2$).

(b) Use the derived formula from part a of this question to find the following integration ($h$=0.2)

$$\int_{0.4}^{1.6} xe^x\,dx$$

0.4 0.6 0.8 1 1.2 1.4 1.6

(10 Marks)

*Good luck to all of you*



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*University of Tripoli*
*Faculty of Engineering*
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# EE 303 Numerical Techniques and Programming

Final Exam, March 8[th], 2012

*a) Answer all questions to the best of your knowledge. (b) Show all steps and carry out all calculations to 4 digits*
*b) No question will be answered during the exam.* *Time allowed:* ***3 hours***

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Q1- For the following linear systems

$$4x + 2y - 10z = -68$$
$$2x - 5y + 12z = 67$$
$$x - 2y + 3z = 14$$

(a) Use LU decomposition to find the inverse of the system (5 Marks)
(b) Solve the system using the inverse found in part (a) of this question. (5 Marks)

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Q2- Use Lagrange interpolating polynomials of the first and second order to estimate f(2.1). Choose the sequence of points from the following table for your estimates to attain the best possible accuracy given that the exact value is 0.3182

| x | 0 | 1 | 2 | 4 | 5 |
|---|---|---|---|---|---|
| f(x) | 2.2000 | 1.0183 | 0.4003 | -2.0000 | -3.8000 |

**(10 Marks)**

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Q3- Evaluate the following integral

$$\int_{0.2}^{0.5} e^{2x} + 2x^2 \, dx$$

(a) Using the trapezoidal rule with n=1. **(2.5 Marks)**
(b) Using Simpson's 3h/8 with h=0.1. **(5 Marks)**
(c) Determine the true percent relative error for each case. **(2.5 Marks)**

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Q4- Given the following ODE

$$\frac{\partial y}{\partial x} = y + 4x^2 \quad , y(0) = 1$$

(a) Find y(1) using 2[nd] order Runge-Kutta (Mid-Point) and Hune's Method with step size 0.25 **(6 Marks)**
(b) What is the relative error in both cases in part (a) of this question **(4 Marks)**

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Q5- Solve the following ODE over the interval x=0 to x=2 with step size of 0.5.

$$6y' - 2y = xy^4 \quad y(0)=-2$$

(a) Using Euler's method **(2.5 Marks)**
(b) Using the classical 4th order RK method (Rengi's Method) **(5 Marks)**
(c) What is the relative error in the answer obtained in part a and b of this question **(2.5 Marks)**

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# Faculty of Engineering
## Electrical and Electronics Engineering Department
### EE 303 Numerical Techniques and Programming
Midterm I, November 2nd , 2009

a) Answer all questions to the best of your knowledge.
b) Show all steps and carry all calculations up to 4 digits unless otherwise mentioned.
c) No question will be answered during the exam.
d) Time allowed: ***2 hours***

Q1- (1) For the function $f(x) = \ln(x)$ , $x > 0$ answer the following:
(a) Find Taylor series expansion.
(b) Derive a recursive formula for the series in part(a)
(c) What is the relative error of 5th order Taylor series expansion of $f(1.1)$ centered around $x$=1.0? (true value=0.09531)
(d) Write a c program for computing the sum of the series. Your program should continue computing until the term value drops down to $10^{-9}$
(e) **Define the following**: ill-condition matrix, Singular matrix, partial and full pivoting, Matrix augmentation.

**(10 Marks)**

Q2- Solve the system

$$2.51\,x + 1.48\,y + 4.53\,z = 0.05$$
$$1.48\,x + 0.93\,y - 1.30\,z = 1.03$$
$$2.68\,x + 3.04\,y - 1.48\,z = -0.53$$

(a) Using Gaussian elimination with out partial pivoting and rounding to 4 decimal places.
(b) Repeat part (a) with partial pivoting and digits chopping after 3 significant figures.
(c) What is the *residual* of the answers obtained in parts (a) and (b) of this question?

**(10 Marks)**

Q3- For the following system

$$\begin{bmatrix} 0.5x & 1.5 \\ 0 & 2.1 \end{bmatrix}$$

(a) Find the value of $x$ that will make the condition number of the system $\cong$100
Your approximation should have error no greater than 0.001

**(10 Marks)**

Good luck to all of you

# Faculty of Engineering
## Electrical and Electronics Engineering Department
### EE 303 Numerical Techniques and Programming
Midterm II, December 14th, 2009

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a) *Answer all questions to the best of your knowledge.*
b) *Show all steps and carry all calculations up to 4 digits unless otherwise mentioned.*
c) *No question will be answered during the exam.*
d) *Time allowed: 2 hours*

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Q1-

(a) Derive Simpson's 1/3 rule by integrating the second-degree Newton-Gregory polynomial that fits $f(x)$ at $x$-values of $x_1, x_2, x_3$ which are evenly spaced at distance $h$ apart. (5 Marks)

(b) Write a C program for computing numerical integration using the trapezoidal method. (3 Marks)

(c) Use *3h/8* Simpson's rule to evaluate the following integral:

$$\int_{-0.4}^{1.4} \sin(2x)\,dx$$

Use $\Delta x$=0.3 (5 Marks)

(d) What is the relative error of the solution obtained in part (C) ? (2 Marks)

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Q2. Using the following data

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| x | -0.243 | 1.123 | 0291 | 0.423 | 0.789 | 1.245 | 2.346 |
| f(x) | 0.8153 | 0.8918 | 1.5877 | 2.1087 | 4.8583 | 12.1478 | 58.2974 |

(a) Using nonlinear least square fitting, find a 3rd degree polynomial and use it to estimate *f(0.5)*. (6 Marks)

(b) Using forward divide difference, find a 5th degree interpolating polynomial and use it to estimate *f(0.5)*. (6 Marks)

(c) Given the true value *f(0.5)*=2.5125, calculate the relative error in both cases and identify which method is more accurate. (3 Marks)

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Good luck to all of you

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# Al-Fateh University

## Faculty of Engineering

### Electrical and Electronics Department

### EE303 - Numerical Techniques and Programming

### Final Examination, January 11th, 2010

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*[A]nswer all questions to the best of your knowledge. No question will be answered during the exam. This [e]xam is closed book and notes.* ***12.5 Marks*** *for each question,* *Time allowed:* ***3 hours***

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[Q]1- Given $\frac{dy}{dx} = y - 2x$ , $y(0) = 1$.

a) What is the relative error obtained in y(2) using Kutta $4^{th}$ order method with h=1. (5 Marks)

b) Compare answers obtained in part a with Mid-Point of $2^{nd}$ order Runge-Kutta. h=0.5. (5 Marks)

c) Write a c program for computing numerical solution to ODE using Huen's method (2.5 Marks)

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[Q]2- Given $y' = 2x^2 - y$, $y(0) = -1$

a) What is the relative error obtained when using Euler's method for evaluating y(1) using h=0.1. (5 Marks)

b) Compare the answer obtained in part a with modified Euler's method. (5 Marks)

c) Write a c program for computing numerical solution to ODE using Euler's method. (2.5 Marks)

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[Q]3- Given

$$\int_{-1}^{3}\int_{0}^{1} e^{-y} \cos(2x)\, dx\, dy$$

a) What is the relative error obtained when using Simpson's $\frac{h}{3}$ rule using $m=n=4$. (10 Marks)

b) Write a c program for evaluating *one dimensional* numerical integration using $\frac{3h}{8}$ Simpson's rule. (2.5 Marks)

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[Q]4.

a) Using Newton-Raphson's method, find the point of interception of the following two functions on (*what is the result when we start with $x_0=0$ and when we start with $x_0=1$ as initial value. Carry calculation until the error drops down to $10^{-4}$, use 6 decimal places*).

$f(x) = 2 + x\cos(2x)$ *and* $g(x) = 2e^{(x-1)}$ (10 Marks)

b) Write a c program for finding roots of non-linear equations using half-interval method (2.5 Marks)

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*Good luck to all of you*



0.918 0.8802 0.8821 0.9198 0.998 1.0.888 1.3628

# Faculty of Engineering

## Electrical and Electronics Engineering Department

### EE 303 Numerical Techniques and Programming

Midterm I, March 28th , 2010

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a) Answer all questions to the best of your knowledge.
b) Show all steps and carry all calculations up to 4 digits unless otherwise mentioned.
c) No question will be answered during the exam.
d) Time allowed: ***2 hours***

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Q1-

(a) Derive the Taylor series at 0 for the function $f(x) = \ln(x+1), x \neq -1$,
(b) Write the series summation notation.
(c) Find a recursive formula of the form $T_n=(\dots)\ T_{n-1}$ for the function in part (a) of this question.
(d) Using f(1.5). How many terms are needed for the absolute error to drop down to $10^{-2}$, given the true value= 0.585
(e) Write a c/c++ program to compute the series in part c of this question

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Q2-

(a) Derive the formula for the secant method
(b) Using Newton-Raphson method, Show that the root of the function $f(x) = \sqrt[m]{N}$ can be written as $x_{n+1} = \dfrac{(m-1)x_n^m + N}{m\ x_n^{m-1}}$
(c) Using the half-Interval method, find the root of $f(x) = 2 * \sin(x) - e^{x/4}$

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**Q3.**

(a) Define the following: Diagonally dominant matrix, ill-conditioned system, partial pivoting, singular matrix.
(b) Give two Matlab commands for finding the condition number of a matrix.
(c) Use ***LU*** factorization method to solve the following system

$$2x + y - 3z = -11$$
$$3x - 4y + 5z = 38$$
$$-2x + 3y + 7z = 15$$

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*Good Luck to all of you*

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# Al-Fateh University, Faculty of Engineering
## Electrical and Electronics Engineering Department
### EE 303 Numerical Techniques and Programming
Midterm II, June 14th, 2009

a) Answer all the questions to the best of your knowledge.
b) Show all steps and carry all calculations up to 3 digits unless otherwise mentioned.
c) No question will be answered during the exam.
d) Time allowed: *2 hours*

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Q1- Using the following data points

| $x_i$ | 1.1 | 2.0 | 3.5 | 5.0 | 7.1 |
|---|---|---|---|---|---|
| $f_i$ | 0.6981 | 1.4715 | 2.1287 | 2.0521 | 1.4480 |

$P_1, P_2, P_3$

(a) Use divide difference to find $f(1.75)$ with polynomials of degrees 1,2, and 3

(b) Given the true value $f(1.75)$= 1.27664, find the relative error in the three polynomials obtained in part (a) of this question.

1.5305

Q2-For the function $f(x) = 2x * \cos(2x)$, find $f'(0.2)$ ~~using a forward-difference~~ approximation, backward difference approximation and central-difference approximation using $\Delta x$=0.1,0.05 and 0.025. Show that the relative error is approximately halved when $\Delta x$ is halved in forward and backward difference while the relative error is approximately quartered when $\Delta x$ is halved in the central difference.

$f'(x) = 2x(-2\sin 2x) + 2(\cos 2x)$ at $x = 0.2$

Q3-

(a) Using 3rd degree Newton-Gregory forward interpolating polynomial that fits four evenly spaced point, derive Simpson's $\frac{3h}{8}$ formula.

(b) Use the formula obtained in part (a) of this question to find the following integral. ($h$=0.2)

$$\int_0^{1.2} \frac{dx}{(x^2+9)^3}$$

(c) Use the trapezoidal rule to find the same integral in part (b) of this question ($n$=6).

(d) Given the true value of the integral=0.001425, which method gave better approximation in term of relative error.

*Good luck to all of you.*

Al-Fateh University, Faculty of Engineering
Electrical and Electronics Engineering Department
EE 303 Numerical Techniques and Programming
Final Examination, July 9th, 2009

*(1) Answer all questions to the best of your knowledge (2) No question will be answered during the exam (3) Show all your steps and carry all computations to 4 decimal places unless otherwise mentioned.*

**(12.5 Marks for each question)**

Q1- Using Simpson's $\frac{1}{3}$ rule for double integration, evaluate the following integral and compare your answer to the analytical solution in term of relative error. ( *Use* $\Delta x=0.3$, $\Delta y=0.2$ )

$$\int_{-0.2}^{1.0}\int_{0.4}^{1.2} e^{x}\cos(2y)\,dy\,dx$$

Q2- Solve $\frac{dy}{dx} = \sin(x) + y$, $y(0) = 2$, analytically then using simplified and improved Euler's methods. Check your answers by finding the relative error to the analytical solution. *Use h=0.25 and tabulate your answers as follows :*

| x | y(x), Simplified Euler | y(x), Improved Euler | y(x), Analytical Answer | Relative Error | |
|---|---|---|---|---|---|
| | | | | Simplified | Improved |
| 0.0 | | | | | |
| 0.25 | | | | | |
| 0.5 | | | | | |
| 0.75 | | | | | |
| 1.00 | | | | | |

Q3- Solve $y' = 2x^2 - y$, $y(0) = -1$, analytically then using second order Runge-Kutta *(Heuns' method)*, compare your answer with the fourth order Kutta's method. *Use h=1 and tabulate your answers as follows:*

| x | y(x) 2nd order | y(x) 4th Order | Analytical Answer | 2nd order Relative Error | 4th order Relative Error |
|---|---|---|---|---|---|
| 0 | | | | | |
| 1 | | | | | |
| 2 | | | | | |

Q4- Given the following matrix

$$\begin{bmatrix} 2.1 & x \\ 3x & 3.5 \end{bmatrix}$$

Find the value of x that will make the condition number of this matrix approximately 100.0. Carry all calculations to 5 decimal places.

*Good luck to all of you.*

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# Al-Fateh University

## Faculty of Engineering

## Electrical and Electronics Engineering Department

### EE 303 Numerical Techniques and Programming

Midterm I, November 25th, 2008

- Answer all questions to the best of your knowledge.
- Programmable calculators are not allowed
- No question will be answered during the exam.

Time allowed 90 *minutes*

Q1-

(a) If the exact answer is A and the approximate answer is $\tilde{A}$, find the absolute and relative error when

1) $A=10.147$, $\tilde{A}=10.159$

2) $A=0.0047$, $\tilde{A}=0.0045$

3) $A=0.671\times 10^{-12}$, $\tilde{A}=0.669\times 10^{-12}$

(b) Find the Taylor series expansion of $f(x)=e^{x^2}$

(c) Find a recursive formula of the form $T_n=(\ldots)\,T_{n-1}$ for the function in part (b) of this question

(d) Write a c/c++ program to compute the series in part c of this question.

**(10 Marks)**

Q2- Use Newton's method to find the intersection of the following two curves

$$x^2+3y^2-1=0$$

$$~~(x-2)^2+(y-1)^2-4=0~~$$

with $x_0=0$ and $y_0=0.5$, perform only 3 iterations.

$x^2-4x+4+y^2-2y+1-4=0$

**(10 Marks)**

Q3- Solve the system of equations:

$$x_1-x_2+2x_3=4$$
$$2x_1+x_2+5x_3=5$$
$$-x_1+4x_2+x_3=-7$$

(a) Using Gaussian elimination with partial pivoting

(b) Show that same answer can be obtained using crammer's rule

**(10 Marks)**

*Good luck to all of you*

$\frac{12x^2}{(2*1)(2x)}$

$(e^x)^2 \quad e^{2x} = e^{2x}\cdot 2 \quad \frac{d}{dx}e^x = e^x\,dx \quad e^{x^2} = f(0) + x f'(0) + \frac{x^2}{2!} f''(0)$

$= 1 + x(0) +$

$\frac{d}{dx}e^{x^2} = e^{x^2}\cdot 2x\,dx \quad \frac{d}{dx}e^{x^2} = e^{x^2}\cdot 2x\,dx$

$f''(x) = e^{x^2}\cdot 2 + 2x\cdot e^{x^2}\,2x\,dx$

$f''(0) = 2$

$e^{x^2}\cdot 4x^2$

$f'''(x) = 2e^{x^2}\cdot 2x\,dx + e^{x^2}\cdot 8x + 4x^2\cdot e^{x^2}2x\,dx$

$f''''(x) = e^{x^2}\cdot 4 + 4x\,e^{x^2}2x + e^{x^2}\cdot 8 + 8x\cdot e^{x^2}2x + 4x^3\cdot e^{x^2}\cdot 2x + e^{x^2}\cdot 8x$

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Al-Fateh University
Faculty of Engineering
Electrical and Electronics Engineering Department
EE 303 Numerical Techniques and Programming
Midterm II, January 7th, 2009

- Answer **any two questions** to the best of your knowledge.
- Carry all calculations to 4 digits
- No questions will be answered during the exam.

Time allowed: 2. hours

Q1- Given a matrix of the form $Ax=b$ where

$$A = \begin{bmatrix} 2.9800 & 1.7600 & -5.0500 \\ 3.7800 & 8.1200 & 0.7600 \\ 2.0223 & 4.3442 & 0.4071 \end{bmatrix}, \quad b = \begin{bmatrix} -300.7336 \\ 2254.2222 \\ 1206.0751 \end{bmatrix}$$

(a) If vector $b$ is changed by (-10%), what is the relative error introduced to the solution vector of this system? (5 Marks)

(b) Given $A = \begin{bmatrix} a & 2 & 2 \\ 2 & 2 & 2 \\ 2 & 2 & a \end{bmatrix}$

(1) What is the value of $a$ that will make this matrix singular? (1 Mark)
(2) What is the value of $a$ that will make the condition number of this matrix > 100? (2 Marks)
(3) Find the Eigen values of the system in term of $a$? (2 Marks)

Q2- For the Following data pairs:

| | | | | | |
|---|---|---|---|---|---|
| $x$ | 0.3 | 0.5 | 0.7 | 0.9 | 1.1 |
| $y$ | 0.40496 | 0.82436 | 1.40963 | 2.21363 | 3.30458 |

(a) Construct a Neville table that interpolates at $x$=0.6 using only the first four points. (3 Marks)
(b) Fit a 3rd degree polynomial of the form $y = a_0 + a_1x + a_2x^2 + a_3x^3$ using Non- linear Least Square and use it to estimate $f(0.6)$ (3 Marks)
(c) Estimate $f(0.6)$ using 3rd degree Newton-Gregory interpolating polynomial (3 Marks)
(d) The table is for $f(x) = x\,e^x$. Which method gave the closest answer to the correct answer. (1 Mark)

Q3- The following table is for $e^{x^2}$

| | | | | | |
|---|---|---|---|---|---|
| $x$ | 0.0 | 0.4 | 0.9 | 1.5 | 1.8 |
| $f(x)$ | 1.0000 | 1.1735 | 2.2479 | 9.4877 | 25.5337 |

(a) Find the first 4 parameters of the fitting polynomial of the form: (3 Marks)

$$P_n(x) = a_0 + \sum_{i=1}^{n} a_i \prod_{i=0}^{n}(x - x_i)$$

(b) Write down the divide difference tables to: (2 Marks)
(1) Estimate $e^{1.2}$ using the first four points (2 Marks)
(2) Estimate $e^{1.2}$ using the last four points (2 Marks)
(c) Write a 4th degree Lagrange interpolating polynomial that interpolates the given data and use it to estimate $f(1.2)$ (2 Marks)
(d) Which method gave the best estimate? (1 Mark)

*Good luck to all of you*

# Al-Fateh University

Faculty of Engineering

Electrical and Electronics Engineering Department

EE 303 Numerical Techniques and Programming

Quiz, December 24th, 2008

• Answer any two questions to the best of your knowledge. Time allowed: **2. hours**

Q1- Given a matrix of the form Ax=b where

$$A = \begin{bmatrix} 3.2000 & -1.0500 & 2.0530 \\ 4.2656 & -1.3997 & 2.7000 \\ -0.0830 & -0.0340 & 1.0470 \end{bmatrix}, \quad b = \begin{bmatrix} -2.2440 \\ -3.0645 \\ 2.0750 \end{bmatrix}$$

(a) Write a c/c++ code for dynamic memory allocation for any matrix A of type float with columns =m and rows=n. (1 Mark)

(b) Write two Matlab. commands for finding the condition number of matrix A. (1 Mark)

(c) Write the formula for the Eigen values of Matrix A(*do not find the values*). (2 Marks)

(d) Using two methods outlined in class, show that the matrix is ill-conditioned. (6 Marks)

Q2- For the Following data pairs:

| | | | | |
|---|---|---|---|---|
| x | 1.2 | 1.8 | 2.5 | 3.6 |
| y | 2.847 | 1.680 | 0.039 | 0.0045 |

(a) Write a Matlab. Command for fitting a polynomial of 3rd degree and for evaluating the same polynomial at x= 2.0. (2 Marks)

(b) Write a Lagrange interpolation polynomial that interpolates the given data and used it to estimate $f(1.5)$ and $f(2.0)$. (4 Marks)

(c) Compare your results with 3rd degree Non-Linear Least Square method. (4 Marks)

Q3- Write down the divide difference table for $e^x$, using the values

| x | $e^x$ |
|---|---|
| 0.0 | 1.0000 |
| 0.4 | 1.49182 |
| 0.9 | 2.4596 |
| 1.5 | 4.4817 |
| 1.8 | 6.0496 |

(a) Find the first 3 parameters of the fitting polynomial of the form: (3 Marks)

$P_n(x) = a_0 + \sum_{i=1}^{n} a_i \prod_{i=0}^{n}(x - x_i)$

(b) Estimate $e^{1.2}$ using (7 Marks)

1-Cubic interpolator with $x_0 = 0.0$

2-Cubic interpolator with $x_0 = 0.4$

Which gives a better estimate?

*Good luck to all of you*

Electrical and Electronics Engineering

Course EE303

Assignment I

Due Date 24/4/2009

1. If the exact answer is A and the computed answer is $\breve{A}$, find the absolute and relative error when

   a) A=10.147, $\breve{A} = 10.159$

   b) A=0.0047, $\breve{A} = 0.0045$

   c) $A=0.671\times10^{12}$ $\breve{A} = 0.669\times10^{12}$

2. Let $a=0.471\times10^{-2}$ $b=-0.185\times10^{-4}$. Use 3 digit arithmetic to compute a+b, a-b, a*b and a/b, find the rounding error and the truncation error if 4 digits were used.

3. Use Taylor series to expand the following functions

$$\frac{1}{x}, \frac{1}{1-x}, \frac{e^x}{\cos(x)}, \log(1-x), \log(1+x)$$

4. The function $f(x) = 2 * \sin(x) - e^{x/4} - 1$ has two roots near x=-5, Use bisection method to find both roots starting with[-7,-5] and [-5,-3]. How many iterations are needed to get the results agree to five significant figures?
   - How much iterations are needed for the absolute error to drop down to $10^{-15}$ in both cases?
   - What will be the relative error after 26 iterations?

5. Using Taylor series derive Newton-Raphson's(N-R) method for finding roots of non-linear equations.

6. Use N-R method to find the roots in function given in question 4. How many iterations are needed to obtain the same results obtained in question 4.

7. What are the advantages and disadvantages of N-R method?

8. Use Newton's method to find the intersection of the curves

$$x^2 + 3y^2 - 1 = 0$$
$$(x-2)^2 + (y-1)^2 - 4 = 0$$

9. Apply secant method for the function in question 4 and compare the results obtained by half-interval, N-R and secant method in term of rate of convergence.

10. Using secant method find where $y = \cos(x)$ and $y = x^3 - 1$ intersect.

# Faculty of Engineering

## Electrical and Electronics Engineering Department

### EE 303 Numerical Techniques and Programming

Answer all questions to the best of your knowledge. Carry all calculations up to 3 decimal places.

**Q1.**

For the function $f(x) = \frac{1}{1-x}$ do the following: **(2.5 Marks each)**

a) Find the Taylor series expansion.

b) Write a recursive formula of the form $T_{i+1} = (\ldots)T_i$

c) Using half-interval method to find the root for $f(x) + 7x^2 = 0$, starting at [1.10, 1.15], continue until the absolute error drops down to $10^{-3}$.

d) What will be the relative error after 36 iterations? How many iterations are needed for the absolute error to drop down to $10^{-19}$?

**Q2.**

a) Using Newton's method, find the root of the following non-linear function **(5 Marks)**

$$f(x) = \tan(x) + 3x^2 - 1$$

Start with $x_0 = 1$

b) Give a definition for each of the following: **( 1 Mark Each)**

1. Partial pivoting and full pivoting.
2. Ill-Conditioned Matrix.
3. Diagonally-Dominant Matrix.
4. Matrix-Augmentation.
5. Condition Number

**Q3.**

a) Find the Eigen Values and the condition number of the following matrix: **(3 Marks)**

$$A=\begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix}$$

*Use Euclidean Norm*

**b)** Use the ***LU*** decomposition to find the solution of the following system **( 7 Marks)**

$$4x_1 - 2x_2 + x_3 = 15$$
$$-3x_1 - x_2 + 4x_3 = 8$$
$$x_1 - x_2 + 3x_3 = 13$$

*Good luck to all of you*

# Al-Fateh University

Faculty of Engineering- Electrical & Electronics Engineering Department

*EE303 Numerical Techniques & Programming* (Final Examination)

*Time Allowed: 3 Hours* (Answer all questions) each question is 10 Marks

Q1. a) Using the Trapezoidal rule, find the numerical integration of the function represented by the following table.

| $x$ | -1.0 | -0.7 | -0.4 | -0.1 | 0.2 | 0.5 | 0.8 | 1.1 | 1.4 | 1.7 | 2. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $f(x)$ | 2.7183 | 1.4092 | 1.0661 | 1.0010 | 0.9920 | 0.8825 | 0.5993 | 0.2642 | 0.0643 | 0.0074 | 0.0003 |

b) Repeat using Simpson's rule.

Q2 - Given the following data

| $x$ | 0.5 | -0.2 | 0.7 | 0.1 | 0.0 |
|---|---|---|---|---|---|
| $f(x)$ | 1.0025 | 1.394 | 1.0084 | 1.0221 | 1.18[illegible] |

Use the divide difference to estimate $f(0.15)$ using a polynomial of degree 3 through the first 4 points

Q3- Use Newton-Gregory forward interpolating polynomial of degree 3 to estimate $f(0.75)$

| $x$ | 0.00 | 0.2 | 0.4 | 0.6 | 0.8 | 1.[illegible] | [illegible].2 | 1.4 |
|---|---|---|---|---|---|---|---|---|
| $f(x)$ | 3.000 | 2.4560 | 1.6880 | 0.7920 | [illegible].1360 | -1.[illegible] | -1.7040 | -2.1520 |

(Hint) $P_n(x_s) = f_0 + \binom{S}{1} \Delta f_0 + \binom{S}{2} \Delta^2 f_0 + \binom{S}{3} \Delta^3 f_0 \ldots\ldots$

Q4 – Using the Simpson's $\frac{1}{3}$ rule for multiple integration, find the value of following

$\int_{[illegible]}^{[illegible]} \int_{0.4}^{2.6} e^{y} \cos(x)\, dy\, dx$

Q5- Estimate the error between $x=1$ and $x=1.6$ when the simple Euler method is used to solve $\frac{dy}{dx} = xy^2$, $y(1)=1$. *The analytical solution is* $y = \frac{3}{3-[illegible]}$

a) Use h=0.1 Compute to the actual errors at each step with the analytical solution.

b) Use Taylor series of order 4 and compare the results with the results obtained in part (a) of this question and the analytical values.

*Good luck to all of you*

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# Alfateh University

## Electrical & Electronics Engineering Department

## EE303 Numerical Analysis

### Mid-Term I

*Answer all questions, Carry calculations to 3 decimal places, time allowed 1.5 hours*

Q(1).

a) Write the Taylor Series Expansion of $f(x)=\sin(x)$ at $x$ near 0

b) Write a recursive expression in the form of $T_{i+1} = (\ldots)T_i$ for the function in part $a$ of this question.

c) Write a *C program* to find the sum of the series obtained in part b of this question

(Q2)

a) Using the Bisection method, find the root of the following non-linear function $f(x) = 2x^3 + 4x^2 - 2x - 5$. Use [1, 2] as starting values. Repeat until the absolute error is <= $10^{-3}$.

b) What will be the *absolute error* after 80 iterations?

c) How iterations are needed for the *relative error* to drop down to $10^{-9}$

(Q3)

The cubic $2x^3 + 3x^2 - 3x - 5=0$ has a root near in [1, 2]. Find the at least three rearrangements that will converge to this root using fixed point- iteration method.

(Q4)

a) Use *Newton's method* to find the root of the following function
$f(x) = \cos(x) - e^{x}$
Start with $x_1 = 1$ and perform only five iterations

b) Repeat part a using the *secant method (only 5 iterations)*. Select any staring values and compare the relative error obtained in both cases.

*Good Luck to All of You*

Alfateh University

Electrical Engineering Department

EE303 Numerical Analysis

## Mid-Term II

*Answer all questions,* *Carry calculations to 3 decimal places,* *time allowed 2 hours*

(Q1) Let $A = \begin{bmatrix} [illegible] & [illegible] & 7 \\ 1 & -8 & 12 \\ 2 & 11 & -1 \end{bmatrix}$, $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ and $b = \begin{bmatrix} 73 \\ 137 \\ -65 \end{bmatrix}$

a) Solve the system using Gaussian elimination without pivoting (5 Marks)

b) By using the Gaussian elimination coefficients find the LU matrices. (5 Marks)

(Q2) Give the following system of linear equations

$12x_1 - 4x_2 + 5x_3 = 59$

$4x_1 - 18x_2 + 2x_3 = 172$

$5x_1 + 14x_2 - 10x_3 = -201$

a) Solve the system using *LU decomposition* with the following two stages (8 Marks)

$Lz = b$ solve for $z$

$Ux = z$ solve for $x$

b) Show that (determinant) $det(A) = det(L) * det(U)$ (2 Marks)

[illegible]

a) Use nonlinear least square fitting to find a polynomial of degree 2 to fit the following tabulated data then find $f(1.0)$ (5 Marks)

| $x$ | -3 | -1.9 | -0.8 | -0.3 | 1.4 | 2.5 | 3.6 | 4.7 | 5.8 |
|---|---|---|---|---|---|---|---|---|---|
| $f(x)$ | 19 | -0.525 | -14 | -21.425 | -22.8 | -18.125 | -7.4 | 9.375 | 32.2 |

b) Fit a fourth order polynomial through the first five points using Lagrangian polynomial and find $f(1.0)$ (5 Marks)

*Good Luck to all of you*

# Alfateh University

## Electrical Engineering Department

## EE303 Numerical Analysis

### Mid-Term I

*Answer all questions, Carry calculations to 3 decimal places, time allowed 1.5 hours*

**Q(1)**

a) Given $\sin(x)=x-(1/3!)x^3+(1/5!)x^5-(1/7!)x^7+\cdots$ Write a recursive expression in the form of $T_{i+1}=(\ldots)T_i$ (3Marks)

b) Write a *C program* to find the root of a nonlinear equation using the secant method. (4Marks)

c) Starting at [7,16], how many iterations are needed for the absolute error to drop down to $10^{-9}$ using the Bisection method (3Marks)

**(Q2)**

a) Use Newton's method to find the root of the following function

$f(x)=\tan(x)-30x$

Start with $x_0=1$ and perform only three iterations (5 Marks)

b) Using Taylor's theorem, show that the error at the $n+1$ iteration can be approximated by: $e_{n+1}\approx\frac{f''(r)}{2f'(r)}e_n^2$. (5 Marks)

**(Q3)** Let $A=\begin{bmatrix}-1 & 4 & -4\\ 2 & 2 & 0\\ 3 & 3 & 2\end{bmatrix}$, $x=\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}$ and $b=\begin{bmatrix}0\\ 1\\ \frac{1}{2}\end{bmatrix}$

a) Solve the system using Gaussian elimination without pivoting and find the L matrix from the steps of Gaussian elimination. (5 Marks)

b) If A can be factorized as $A=LU$, solve the system $Ax=b$ using the following two stages

$Lz=b$ solve for $z$

$Ux=z$ solve for $x$ (5 Marks)

# Faculty of Engineering
## Electrical and Electronics Engineering Department
### EE 303 Numerical Techniques and Programming

Answer all questions to the best of your knowledge. Carry all calculations up to 3 decimal places.

*Time allowed: 2 hrs.*

Q1.

a) Find the Taylor series expansion for the function $f(x) = \ln\left(\frac{1+x}{1-x}\right)$ **(2.5 Marks)**

b) Write a recursive expression of the form $T_{i+1} = (\ldots)T_i$ for the Taylor series expansion of the function given in part a of the question. **(2.5 Marks)**

c) Apply Newton's method to equation $x^2 = N$ to derive the algorith for getting the square root of N. **(2.5 Marks)**

d) Write a c program to find the root of a non-linear equation using secant method. **(2.5 Marks)**

Q2.

Use Newton's method to find the roots of the following non-linear system

$f(x,y) = 4 - x^2 - y^2$

$g(x,y) = 1 - e^x - y$

Start at $x_0 = 1, y_0 = -1.7$, perform only 3 iterations. **(10Marks)**

Q3.

Given

$$A=\begin{bmatrix} -4 & -12 & 3 \\ 10 & -4 & 5 \\ -5 & 2 & 6 \end{bmatrix}, b=\begin{bmatrix} -46 \\ 4 \\ 15 \end{bmatrix}$$

a) Solve the system using Gaussian elimination method with partial pivoting. **(5 Marks)**

b) Show that same solution can be obtained using Cramer's rule. **(5 Marks)**

Q4.

For the matrix given in q3.,

a) Show that *det(A)=det(L)*det(U)* **(5 Marks)**

b) Show that the matrix is not *ill-conditioned.* (*Use Euclidean Norm*) **(5 Marks)**

*Good luck to all of you*

# Al-Fateh University

## EE303 Numerical Techniques & Programming

### Final Examination

*Time Allowed: 3 Hours* (Answer all questions) each question is 10 Marks

Q1. Using Newton's method, find the root of the following non-linear function $f(x) = e^{-x^2} - \sin(2x)$ in the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ repeat until the error $\leq 10^{-5}$

Q 2. Using non-linear least square fitting, find a polynomial of degree 3 to fit the following data

| X | 0.5000 | 0.8000 | 1.0000 | 1.3000 | 1.6500 | 2.0000 | 2.3000 | 3.0000 |
|---|---|---|---|---|---|---|---|---|
| F(x) | -2.6500 | 0.3920 | 2.9000 | 7.7420 | 15.6091 | 26.6000 | 39.0920 | 82.1000 |

b) Find the value of the function at x=1.5

Q3. a) Using the Trapezoidal method, find the numerical integration of the function represented by the following table.

| X | -1.0 | -0.7 | -0.4 | -0.1 | 0.2 | 0.5 | 0.8 | 1.1 | 1.4 | [illegible] | [illegible] |
|---|---|---|---|---|---|---|---|---|---|---|---|
| F(x) | 2.7183 | 1.4092 | 1.0661 | 1.0010 | 0.9920 | 0.8825 | 0.5993 | 0.2642 | 0.0643 | 0.0074 | 0.00[illegible]3 |

b) Repeat using Simpson's rule

Q.4 Use composite Simpson's rule to approximate the following multiple integrals

$$\int_{1.4}^{2.0}\int_{1.0}^{1.5} \sin(x+2y)\, dy\, dx$$

with n=m=4

Q5. The following ordinary differential equation

$y' = y - t^2 + 1, \quad 0 \leq t \leq 2, \quad y(0) = 0.5$ has an exact solution $y(t) = (t+1)^2 -$ [illegible]

a) Use Euler's method with N=10 to approximate the solution

b) Use Taylor series of order 4 and compare the results with the results obtained in part a) of this question and the analytical values

*Good Luck to All of You*